Theoretical Models of Vote Buying

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Outline

- Part 1 : Lobbying, Vote Buying : Generalities, Cases and Empirical Evidence
- Part 2 : A General Influence Buying Model and its Main Special Cases
- Part 3 : Sequential Competitive Vote Buying with Procedural Legislators
- Part 4 : Sequential Competitive Vote Buying with Consequential Legislators

Lobbying (I)

- Lobbying: **Buying access or buying influence** (votes, agenda setting, draft of bills,...). Understanding the positive and normative implications of lobbying mechanisms and activities on the determination of economic and social policies is an important part of the research agenda of modern political science and political economics. This scientific knowledge is useful in assessing the role of political institutions (elections, legislatures,..) and actors on the prevalence of these activities and the design and evaluation of reforms of these institutions
- Legality : Lobbying defined as buying access to politicians and decision-makers is considered to be legal. It is however recognized that they need to be regulated and be very transparent.

Lobbying (II)

- Influence/Vote buying : you buy the vote of a person (voter in a general election, legislator, member of a committee/jury,...) via different transaction technologies (cash, job promises, local public goods, campaign contributions, presents and holidays,...) which are more or less efficient. The contract can be sophisticated and conditional upon contingencies verifiable by both parties. We will see several practices of vote buying.
- It is however illegal : **Constitution du 5 fructidor an III (22 août 1795)**

Article 32. - Tout citoyen qui est légalement convaincu d'avoir vendu ou acheté un suffrage, est exclu des Assemblées primaires et communales, et de toute fonction publique, pendant vingt ans ; en cas de récidive, il l'est pour toujours.

Lobbying (III)

- Campaign contributions : interest groups that donate money to politicians can alter the outcome of legislation either by gaining influence over how representatives act while they are in office (influence motive) or by affecting which candidate win election to the legislature (electoral motive).

Lobbying (IV)

- A Figure

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History of Thought (I)

- History of thought: Political scientists and economists have always been intererested by these topics (Olson, Stigler, Tullock to cite a few). They were interested in questions like : Is lobbying a wasteful (unproductive) or useful (revealing intensity of preferences) activity ? How do we measure the deadweight loss resulting from these activities ? (Tullock famous rectangle) What makes a special interest group (social group, economic sector) more or less efficient in influencing the social outcome (Olson's program on the determinants of the success of collective action)?
- Strong impact of Grossman-Helpman (1994)'s paper "Protection for Sale". There is a deviation with respect to the Olsonian agenda. GH's main concern is positive instead of normative: to evaluate the importance of lobbying in explaining the differences in protection across sectors. It has generated an important theoretical and empirical literature.

History of Thought (II)

- The difference between access and influence has been defined from a theoretical perspective. Buying access opens the possibility of communicating information to the decision makers and try to persuade the latter that their preferred positions would also serve the policy makers' interests. GM (2001)'s book is divided in two parts according to these two angles. Few citations
- "A third possibility, and the most invidious, is that contributions buy influence. This, of course, is the view that many social scientists, politicians, and media persons, and it has spawned popular demands for campaign finance reform. Influence can come at any stage of the legislative process. "The payoff may be as obvious and overt as a floor vote in favor of the contributor's desired tax loophole or appropriation" writes William Proxmire, a former senator from Wisconsin. But this is not the only possibility...

History of Thought (III)

- Documenting that money affects policy outcomes has been not easy task. After all, it is difficult to know what a bill would have looked like absent the net effect of all contributions. Even, if we focus on roll call votes as many researchers have done, the effort is confounded by the counterfactual: how would a legislator have voted absent the contributions ?...
- The link between a contribution and a legislator's actions need not be made explicit. Indeed, most elected officials would rankle at the suggestion that legislative favors are being provided in exchange for campaign gifts. But from repeated interaction with a lobbyist, a legislator may come to recognize when such a link exists, and may learn to interpret the lobbyist's code words that indicate how important is an issue to the group. As former congressman Tim perry of Minnesota succintly put it, "There's no tit for tat in this business, no check for a vote. But notheless the influence is there. Candidates know where their money is coming from"

History of Thought (IV)

- (GM, Chapter 7, Buying Influence, pp 225-226) : "We have seen how interest groups can use their knowledge of the issues and candidates as a tool for poltical influence. Our focus swithches now from information to money.....SIGs play a large and growing role in political financing.....The PACs contributed almost one-third of the funds raised by candidates for the U.S. Congress and accounted for a goodly share of the revenues of the political parties.

History of Thought (V)

- What do the interest groups buy with their contributions? This is a sensitive question and one that generates considerable controversy. The groups themselves claim that their giving is intended only to secure access to the policymakers...But most observers do not believe that access is the groups' only aim...The suggestion that SIGs are trading money for influence conjures up a sordid image. Some readers might envision corrupt politicians in smoke-filled rooms peddling favors and haggling over the price. But the reality can be quite different from this. The interaction between SIG and policymaker need not involve any explicit discussion of a quid pro quo. Rather, influence can be bought and sold by a subtle exchange in which both sides recognize what is expected of them. The SIG can make known by its words and deeds that it supports politicians who are sympathetic to its cause. then the policymaker can appear to be taking actions to promote a constituent's interests while gratefully accepting

Few Figures (I)

- Lobbying is a significant activity in all countries (US, Europe but also developing countries) even if (according to some scholars) we would expect to see more money to be invested in politics. See GH (2001) for figures indicating on one hand the number and levels of activity of the lobbying firms, the lobbying related expenses of organizations and campaign giving by special interest groups through political action committees (PAC)
- More regulated activity: Lobbying Disclosure Act of 1995 (US).

Few Figures (II)

- Europe: About 3000 interest groups in the European Union (600 regional or national have an office in Brussels, 200 firms and more than 500 "cabinets-conseils". There are about 10000-15000 lobbyists in Brussels in front of 25000 European bureaucrats and 626 legislators.
- France: There are 51 firms and cabinets providing lobbying services to clients (two federations l'Association Francaise des Conseils en Lobbying founded in 1991 and L'Association francaise des Conseils en Affaires publiques founded in 2001). The reglementation has changed (Resolution N° 3399 in November 2006).

Vote Buying in General Elections (I)

- Well documented (International associations fighting against corruption and for more transparency, Trials,...)
- Open voting versus secret voting. A classification of vote buying strategies (positive versus negative vote buying)
- Overview of various vote buying techniques (Shaffer and Sheder (2007) "Elections for Sale"). Enforcing and monitoring ("the Carousel")
- Latin America: Vote buying practices of Peronists in Argentina (Stokes (2005), Nichter (2008)); Brazil (in 2001 municipal elections 7% of the voters were offered money for their votes); Mexico (Different surveys in Mexico places the frequency of vote buying as between 5% and 26%).

Vote Buying in General Elections (II)

East Asia: Philippines (7% of the all voting-aged adults were offered some form of payment in community level elections); Thailand (30% of household heads surveyed in a national sample were offered money in the 1996 general election); In Taiwan's third lagest city, 27% of the random sample of eligible voters reported in 1999 that they accepted cash during previous campaigns). The amount of money offered to voters varies gretly depending on the competitiveness of the election and local levels of prosperity. Personalized networks of support with the "Vote Brokers" as the key players.

Vote Buying in General Elections (III)

Table 1 from Gans and Co

Vote Buying in General Elections (IV)

Table 2 from Gans and Co

Vote Buying in General Elections (V)

The tract from Sao Tome and Principe (Field Experiment conducted by Vincente (2008) in this small country of West Africa). He identifies vote buying patterns and finds evidence that vote buying disproportionately benefits well-fianced challengers and that voter information campaigns can be effective in reducing vote buying.

Vote Buying in legislatures (I)

Illustration from the Old Times

"The bribing of legislatures and other decision making bodies for the furtherance of special interests has a long history that doubtless has not ceased to the present day. One of the most notorious alleged cases of bribary was reported by the Scotsman George Lockhart in his Memoirs of the Affairs of Scotland. Lockhart charged that the treaty of Union, which created Great Britain in 1707, was achieved by the Queen's minister selectively bribing members of the Scottish Parliament. The most astonishing aspect of Lockhart's account is that he published, in 1714, alist of the members bribed and the prices paid. the list contains 32 names, including the Duke of Queensberry, who allegdly got £12325, the Earl of Marchmont £1104, the Marquis of Tweeddale £1000, and so on, the lowest man going for £11. All of those allegedly bribed except one (the Duke of Atholl) voted for the union" (From Young (1978)).

Vote Buying in legislatures (II)

Illustration from the Present Days (I)

- It is never easy to provide evidence of vote buying (Scandals, suspicions, statistical analysis,...)
- Recent Parliamentary scandal in Britain .The Britain's House of Lords voted to suspend two of its members. It's first time a lord has been barred since 1642. Lord Taylor of Blackburn and Lord Truscott allegedly agreed to use their influence to get a loophole into a proposed tax law in exchange for cash, but the person offering the bribe turned out to be an undercover reporter. The Sunday Times newspaper ran the sting operation, in which journalists approached members of the House of Lords, claiming to be acting on behalf of a foreign business looking to open a chain of stores in the United Kingdom.

Vote Buying in legislatures (III)

Illustration from the Present Days (II)

- Secret recordings showed Lord Truscott, a former energy minister, discussing a fee of up to $113,000 to work 'behind the scenes' on behalf of the lobbyists, while Lord Taylor offered to conduct a campaign to persuade ministers and officials for a one-year retainer of $189,000," the newspaper said.
- The code of conduct for the House of Lords states that members "must never accept any financial inducement as an incentive or reward for exercising parliamentary influence." But peers are unelected, don't receive a salary, and are free to pursue outside income; around 140 have "outside consultancies." There are few sanctions against peers who transgress the rules: they cannot be removed from the house, but merely "named and shamed." That, says Baroness Royall, the Labour leader of the Lords, is "bananas." It's hard to disagree. MPs convicted of criminal offenses or found to

Vote Buying in legislatures (IV)

- The peers identified by the Sunday Times deny acting improperly, and Lord Truscott has said he knew the supposed lobbyist was an undercover reporter and was simply stringing him along. In video footage posted on the Sunday Times website, Truscott boasts of his connections and explains what his work would entail: "It is. . .identifying people. . .meeting people, talking with people to facilitate the amendment and making sure the thing is granted," he says. "I think the other thing is identifying. . .who can be approached to put forward amendments at various stages and maybe other bodies to contact." In an audio clip, his colleague Lord Taylor tells an undercover reporter that "some companies that I work with would pay me £100,000 a year" to facilitate meetings with decision-makers. When questioned by the reporter about that fee, Taylor says: "That's cheap for what I do for them. . .I am not exaggerating. It's whether I want to do it or not. You've got to whet my appetite to get me to come on board."

Vote Buying in legislatures (V)

Illustration from the Present Days (III)

- A measure explicitely introduced to counteract vote buying was the imposition of the secret ballot. The effectiveness of the secret ballot as a deterrent to vote buying is debatable. Several studies indicate that the process of vote buying has simply hsifted from simple schemes to more intricate ones (Cox and Kousser (1981) trace the effects of the imposition of the secret ballot on vote buying in New York elections. They find that the secret ballot substantially changed the form but not necessarily the amount of vote buying)
- Taiwanese Presidential Election of 2000 : the ruling National Party subsidized betting parlors to offer extremely favorable odds on the event that the party candidate, Lien Chan, was elected

Vote Buying in legislatures (VI)

- Salt Lake City 2002 Olympic Winter Games: certain members of the International Olympic Committee (IOC) were paid money in exchange for their votes, as well as a bonus conditional on the outcome of the vote.
- Such sophisticated vote buying contracts, where payments are contingent on an individual's vote as well as some aggregate measure can be found as far back as nineteenth century great Britain. Seymour details how in elections held in Liverpool in the 1830s the price paid for votes rose and fell like a stock price, depending on the current vote share of the candidates. The price was often posted openly outside the the polling station and updated numerous times during the curse of the day.
- Vote buying at the International Whaling Commission (IWC)

Vote Buying in legislatures (V)

Illustration from the Present Days (IV)

- A Case Study (The October 2004 Quarterly Report of the Tobbaco-Free Kids Action Fund). The report provides detailed updates of the tobacco industry's campaign contributions to federal candidates, political parties and PAC....There has been a significant amount of activity during this session of congress regarding legislation to grant the FDA regulation to regulate tobacco products. For the first time in history, a House of Congress, the U.S. Senate, voted to pass FDA regulation, and did so overwhelmingly. The senate passed amendment, sponsored by senators Mike DeWine (R-OH) and Edward Kennedy (D-MA), included FDA regulation of tobacco and a buyout of tobacco farmers.

Vote Buying in legislatures (VI)

- It was included in the Senate version of a large corporate tax bill called the Foreign sales Corporation, or FSC bill. The house version of the bill included a tobacco buyout, but not FDA regulation...As detailed later in this report, those members of the confrence committee who voted against the inclusion of FDA regulation have recently received, onaverage, about five times the amount of the tobacco industry contributions as their counterparts who supported the FDA provision".
- The appendix C of the report is hallucinating !
- Many econometric models have been developed to evaluate if the vote of legislators on specific issues (for instance trade and defense) responded to the amount of contributions that they received from business and labor interests.

Part 2

A GENERAL MODEL OF BUYING INFLUENCE

General Model (I)

- $M + N$ players: M lobbyists and N public decision-makers
- Each decision maker n has his own sphere of action (influence) described by a feasible set of actions A_n.
- The profile of actions $a = (a_1, a_2, ..., a_N)$ generates through an outcome function H a social outcome $x = F(a)$. The set X of feasible social outcomes is the range of H i.e. $H(A)$ where $A = \prod_{n=1}^{N} A_n$
- The gross payoff of lobbyist m is described by the utility function $W^m : X \to \mathbb{R}$
- The gross payoff of decision-maker n is described by the utility function $V^n : X \times A_n \to \mathbb{R}$

General Model (II)

- Their net payoffs are defined after that monetary transfers will be introduced. The game goes as follows.
- Each lobbyist m makes an offer $T_n^m \geq 0$ to decision maker n : details of the game will depend upon the precise spectrum of offers (conditional upon x or(and) a_n or flat). An offer to decision-maker n is therefore a function $T_n^m : X \times A_n \longrightarrow \mathbb{R}_+$. A strategy for lobbyist m is a N-dimensional vector of individual offers $T^m = (T_1^m, T_2^m, ..., T_N^m)$. In addition to the restrictions on individual offers which have been mentioned above, we will consider sometimes restrictions on the set of strategies like for instance the impossibility to make different individual offers.

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General Model (III)

- Once the first stage of the game has been completed i.e. when each decision-maker has learned the offers that he has received from the diffferent lobbyists, the second stage consists of a continuation game between social decision makers. A strategy for decision maker n is an action $a_n \in A_n$.
- Given the profile of strategies $\left(T^1, T^2, ..., T^M, a_1, a_2, ..., a_N\right)$, the payoff of:

· Lobbyist m is $W^m\left(F(a)\right) - \sum_{1 \leq n \leq N} T_n^m$

· Decision-maker n is $V^n\left(F(a), a_n\right) + \sum_{1 \leq m \leq M} T_n^m$

Canonical Special Cases

- Deterministic Common Agency (Grossman-Helpman 1994)
- Rent Seeking (Tullock 1975)
- Stochastic Common Agency
- Vote Buying and Legislative Lobbying
  - Multi-Unit First Price and All-Pay Auctions
  - Colonel Blotto games
  - Sequential Lobbying (Groseclose-Snyder 1996)

Deterministic Common Agency (I)

- $N = 1$, M arbitrary
- $X = A$
- $T^m : A \to \mathbb{R}_+$
- Optimal response of the decision-maker (no continuation game)
- Main Result 1 : Coalition-proof Nash equilibria exist and induce action(s) a which are efficient i.e. which maximize the social surplus $\sum_{1 \leq m \leq M} W^m(a) + V(a)$ (Bernheim and Whinston (1987))

Deterministic Common Agency (II)

- Main Result 2 : Characterization of the payoffs of the players. A vector $\left(W^{1*}, W^{2*}, ..., W^{M*}\right)$ is a vector of equilibrium payoffs of the lobbyists in this game iff it a Pareto efficient vector of the polyhedron

$$\left\{ W \in \mathbb{R}^M : \sum_{m \in S} W^m \leq \pi(\{1, 2, ..., M\}) - \pi\left(\{1, 2, ..., M\} \setminus S\right) \right\}.$$

where π is the TU cooperative game defined as follows on the set $\{1, 2, ..., M\}$

$$\pi(S) = \underset{a \in A}{Max} \sum_{m \in S} W^m(a) + V(a) \text{ for all } S \subseteq \{1, 2, ..., M\}$$

where V is normalized such that $\underset{a \in A}{Max} V(a) = 0$

Deterministic Common Agency (III)

- The payoff V^* of the decision maker is $\pi(\{1, 2, ..., M\}) - \sum_{1 \leq m \leq M} W^{m*}$ (called the *rent of the politician* by Laussel and Le Breton (2001) as this payoff arises purely from the existence of the competition between the lobbyists)
- Laussel and Le Breton provide sufficient conditions on the common agency game for the rent V^* to dissapear and sufficient condition for the equilibrium to be unique. The amount of V^* is a good proxy for the intensity of the competition between the lobbyists: V^* is the aggregate amount of money received by the politician called *measure of societal conflict* by Esteban and Ray (1999).

Deterministic Common Agency (IV)

- GM have considered the case where the decision a is a one dimensional variable describing the protection of a domestic sector against foreign competition. The society is partionned into L groups (special interests) out of which M are represented by M lobbyists. Some of these groups may support protection while others may oppose.
- The gross aggregate payoff of group $m = 1, ..., M, M+1, ..., L$ is denoted $W^m(a)$.
- The payoff of the politician is:

$$V(a) = \alpha \sum_{1 \leq m \leq L} W^m(a)$$

where α is a non negative parameter

Deterministic Common Agency (V)

- From what precedes, at equilibrium a^* maximizes:

$$(\alpha+1)\sum_{1\leq m\leq M} W^m(a)+\alpha\sum_{M+1\leq m\leq L} W^m(a)$$

or equivalently

$$\sum_{1\leq m\leq M} W^m(a)+\frac{\alpha}{\alpha+1}\sum_{M+1\leq m\leq L} W^m(a)$$

Deterministic Common Agency (VI)

- If $L = M$ i.e. if all the groups are involved into lobbying then a^* is socially efficient. Also, if α is large, then a^* is almost efficient. Otherwise, i.e. if not all the groups are represented in society and if the concern of the politician about social welfare is moderate (in the extreme case, if $\alpha = 0$), then a^* will depart from efficiency.
- In GM, each active group, on its own, faces a collective action problem and may anticipate free riding issues: a contributor to a lobby faces a public good provision problem in a setting where each group competes with the other groups. These issues which are essential in Olson's program are ignored by GM. LLB show that at equilibrium free riding vanishes.
- Empirical testing of GM and variants. What is the value of α ?

Rent Seeking (I)

- $N = 1$, M arbitrary
- $X = A = \left\{a^1, a^2, ..., a^M\right\}$: a decision is here a prize allocated to a single lobbyist.
- $W^m(a^m) \equiv W^m > 0$ and $W^m(a^k) = 0$ for all $k \neq m$
- $V \equiv 0$
- $T^m \in \mathbb{R}_+$
- The reaction of the politician is modeled as a stochastic response

Rent Seeking (II)

- Probability of selecting action a^m is

$$\frac{(T^m)^r}{\sum_{1\leq k\leq M}(T^k)^r}$$

where r is a positive parameter

- Two special cases:

$r = 1$ (*the pure lottery model*)
$r \longrightarrow \infty$ (*the all-pay auction model*) where the probability of selecting action a^m is equal to $\frac{1}{K}$ if $m \in \underset{1\leq k\leq M}{ArgMax}\, T^k$ (where $K = \left|\underset{1\leq k\leq M}{ArgMax}\, T^k\right|$) and to 0 otherwise

Rent Seeking (III)

- In this model, the structural decision model of the politician is not described. The payments of the lobbyists are unconditional.
- When $r \longrightarrow \infty$, there is a no equilibrium in pure strategies and a continuum of equilibria in mixed strategies all generating the same revenue for the politician (Baye, Kovenock and de Vries, 1993, 1994, 1996)
- When $r = 1$, there is a unique equilibrium in pure strategies (Hillman and Riley (1989), Fang (2002)).

Rent Seeking (IV)

- Let the lobbyists be ordered according to their valuations i.e: $W^1 \geq W^2 \geq ..... \geq W^M$. It can be demonstrated that the politician may find profitable to exclude some high valuation lobbyists from the competition in the all pay auction but this is never the case in the lottery model.
- Esteban and Ray (1999) have generalized the lottery model to the case where the matrix $W = (W_k^m)_{1 \leq k,m \leq M}$ is not necessarily diagonal

Stochastic Common Agency (I)

- An unexplored new territory for a PhD Project !!! Reconcile common agency and rent seeking.
- $N = 1$, M arbitrary
- $X = A = \left\{a^1, a^2, ..., a^M\right\}$: a decision is here a prize allocated to a single lobbyist.
- $W^m(a^m) \equiv W^m > 0$ and $W^m(a^k) = 0$ for all $k \neq m$
- $V(a^m) \equiv \varepsilon^m$ for all $m = 1, ..., M$; the vector $\left(\varepsilon^1, \varepsilon^2, ..., \varepsilon^M\right)$ is a random vector private information of the politician
- $T^m = T^m(a^m) \in \mathbb{R}_+$

Stochastic Common Agency (II)

- The payoff of the politician when he selects a^m is $\varepsilon^m + T^m$
- The expected payoff of lobbyist m is (Probability that a^m is selected) $\times (W^m - T^m)$
- Probability that a^m is selected $=$ Probability$\left(\varepsilon^m - \varepsilon^k > T^k - T^m \text{ for all } k \neq m\right)$
- Illustration when $M = 2$ and ε^1 and ε^2 are iid random variables with cumulate F and density f (Probit and Logit)

Vote Buying, Legislative Lobbying (I)

- $M = 2$, N arbitrary (the social decision makers are here legislators; alternatively, we can interpret the N members of as voters, districts of voters,.....)
- $X = \{x^1, x^2\}$
- $W^m(x^m) \equiv W^m > 0$ and $W^m(x^k) = 0$
- $A_n = \{x^1, x^2\}$: legislator n either supports policy x^1 or supports policy x^2 (only two votes, no abstention)
- $H(a) = x^2$ iff $\{n = 1, ..., N : a_n = x^2\} \in \mathcal{W}$ where $\mathcal{W}$ is a monotonic family of subsets of $\{1, 2, ..., N\}$

Vote Buying, Legislative Lobbying (II)

- $(N, \mathcal{W})$ is called a simple game, voting game, committee ($\mathcal{W}$ is the set of winning coalitions)
- Two alternative behavioral assumptions

$$\begin{cases} \text{Procedural Legistors } V_n(x, a_n) = v_n(a_n) \\ \text{(they care about the way they vote)} \\ \text{Consequential Legistors } V_n(x, a_n) = v_n(x) \\ \text{(they care about the outcome)} \end{cases}$$

Multi-Unit First Price and All-Pay Auctions (I)

- **Simultaneous** vote buying without budget constraints. Auctions of several objects whose value arises if they come together in appropriate combinations.
- Only the case of the majority game: N is odd and $S \in \mathcal{W}$ iff $|S| \geq \frac{N+1}{2}$ has been considered

Multi-Unit First Price and All-Pay Auctions (II)

- Some alternative payoff functions have been considered. For instance, the game where $X = A = \prod_{n=1}^{N} A_n$ and $W^m(x) = \sum_{1 \leq n \leq N} w_n x_n^m$ where $x_n^m = 1$ if $a_n = x^m$ and 0 otherwise and $w_n > 0$ is an exogeneous parameter has received a lot of attention. In voting, it corresponds to the situation where lobbyists care about the number of legislators or voters who voted for them where each voter is weighted by a parameter which can receive different interpretations. In auctions, it corresponds to multi-unit selling without synergies.
- The analysis of these games is very tricky. If the payment of legislator n is only based on a_n, then we have a first price auction (Szentes and Rosenthal (2003), Szentes (2007)). We could also consider payment schemes contingent upon the outcome

Colonel Blotto Games (I)

- Often the valuations are ignored; implicitely either the budget constraints are active or the opportunity cost of money is simply ignored. Let B^m be the budget of lobbyist m.
- We obtain then a Colonel Blotto game where the set of strategies of each lobbyist is determined by its budget and the constraints on the payments that can be made to the legislators. Only the majority game has been analysed. Further, depending upon the specification of payoff, different variants can be considered. Two are prominent : either to win a majority (Majority) or to win the largest number of votes (Plurality, possibly weighted)
- All specifications lead to a two-player zero-sum game highly discontinuous

Colonel Blotto Games (II)

- Existence and Partial Characterization of equilibria in mixed strategies (Borel (1921), Gross and Wagner (1950), Hart (2008), Kvasov (2007), Laslier and Picard (2002), Myerson (1993), Robertson (2006), Thomas (2008), Young (1978), Weinstein (2005)).
- Relationships with campaign spending games. Generalizations to the case where there are more than one input (medium of exchange) to buy the votes of a district (public jobs, favors, presents,...)
- Smoothing the discontinuities through probabilistic response : swing voters and partisans (Dixit and Londregan (1996), Laslier and Le Breton (2000)).

Colonel Blotto Games (III)

- How the value of the plurality Colonel Blotto game varies with the ratio $\frac{B^2}{B^1}$?
- How the value of the majority Colonel Blotto game varies with the ratio $\frac{B^2}{B^1}$?
- The majority game is symmetric : when $\frac{B^2}{B^1} = 1$, **the price received by any legislator** is drawn randomly over $\left[0, \frac{2}{N} B^m\right]$
- When legislator n has weight w_n and $\frac{B^2}{B^1} = 1$, the the price received by legislator n in the plurality Colonel Blotto game is drawn randomly over $\left[0, \frac{2w_n}{\sum_{1 \leq j \leq N} w_j} B^m\right]$
- Many open problems

Part 3

SEQUENTIAL COMPETITIVE VOTE BUYING WITH PROCEDURAL LEGISLATORS

Outline

- Introductory Remarks
- Ingredients
- Victory Threshold
- Connections to Hypergraphs: Fractional and Covering Numbers
- Nucleolus and Least Core
- Weighted Majority Games
- Real World Illustrations
- Optimal Hurdle Factor
- Buying Supermajorities