Majority Judgment vs Majority Rule

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Based on joint work with

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Actual rules in diving

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  - $2\frac{1}{2}$ to $4\frac{1}{2}$ "deficient"
  - 5 to 6 "satisfactory"
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- There are many other instances that use measures—well defined scales of grades—to grades, to rank and or to designate winners : guide Michelin, figure skating, gymnastics, concours Chopin, wine competitions, etc.

A real use of Majority Judgment : small jury

Opinion profile : LAMSADE Jury ranking PhD candidates for a grant, 2015

	J_1	J_2	J_3	J_4	J_5	J_6
A :	*Excellent*	*Excellent*	*V. Good*	*Excellent*	*Excellent*	*Excellent*
B :	*Excellent*	*V. Good*	*V. Good*	*V. Good*	*Good*	*V. Good*
C :	*Passable*	*Excellent*	*Good*	*V. Good*	*V. Good*	*Excellent*
D :	*V. Good*	*Good*	*Passable*	*Good*	*Good*	*Good*
E :	*Good*	*Passable*	*V. Good*	*Good*	*Good*	*Good*
F :	*V. Good*	*Passable*	*Insufficient*	*Passable*	*Passable*	*Good*

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Merit profile :

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D :	*V. Good*	*Good*	*Good*	*Good*	*Good*	*Passable*
E :	*V. Good*	*Good*	*Good*	*Good*	*Good*	*Passable*
F :	*V. Good*	*Good*	*Passable*	*Passable*	*Passable*	*Insufficent*

A succinct, simple description of MJ

	Excellent	*Very Good*	*Good*	*Passable*	*Insufficient*
A :	5	1			
B :	1	4	1		
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Merit profile (counts), LAMSADE Jury.

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Ranking PhD candidates B and C by LAMSADE Jury :

B :	*Excellent*	*V. Good*	V. Good	V. Good	*V. Good*	*Good*
C :	*Excellent*	*Excellent*	V. Good	V. Good	*Good*	*Passable*

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C :	**Excellent**	V. Good	V. Good	**Good**

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For all pairs (except between B and C), first order domination decides!

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	A	C	B	D	E	F	Borda score
A	–	5	5	6	5.5	6	5.5
C	1	–	3.5	5	4	5	3.7
B	1	2.5	–	5.5	5	6	4.0
D	0	1	0.5	–	3.5	5	2.0
E	0.5	2	1	2.5	–	4	2.0
F	0	1	0	1	2	–	0.8

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Condorcet ranking is $A \succ_{Condo} C \succ_{Condo} B \succ_{Condo} D \succ_{Condo} E \succ_{Condo} F$.

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This is a major criticism against MJ (and Borda since the 18th century).

The majority judgement ballot (large electorate)

Ballot : Election of the President of France 2012

To be president of France,
having taken into account all considerations,
I judge, in conscience, that this candidate would be :

	Outs-tanding	*Excel-lent*	*Very Good*	*Good*	*Accep-able*	*Insuf-ficient*	*Reject*
François Hollande							
François Bayrou							
Nicolas Sarkozy							
Jean-Luc Mélenchon							
Nicolas Dupont-Aignan							
Eva Joly							
Philippe Poutou							
Marine Le Pen							
Nathalie Arthaud							
Jacques Cheminade							

Pool OpinionWay-Terra Nova, April 12-16 2012

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Hollande	12.48%	16.15%	16.42%	11.67%	14.79%	14.25%	14.24%
Bayrou	2.58%	9.77%	21.71%	25.24%	20.08%	11.94%	8.69%
Sarkozy	9.63%	12.35%	16.28%	10.99%	11.13%	7.87%	31.75%
Mélenchon	5.43%	9.50%	12.89%	14.65%	17.10%	15.06%	25.37%
Dupont-Aignan	0.54%	2.58%	5.97%	11.26%	20.22%	25.51%	33.92%
Joly	0.81%	2.99%	6.51%	11.80%	14.65%	24.69%	38.53%
Poutou	0.14%	1.36%	4.48%	7.73%	12.48%	28.09%	45.73%
Le Pen	5.97%	7.33%	9.50%	9.36%	13.98%	6.24%	47.63%
Arthaud	0.00%	1.36%	3.80%	6.51%	13.16%	25.24%	49.93%
Cheminade	0.41%	0.81%	2.44%	5.83%	11.67%	26.87%	51.97%

Majority Grade et Gauge

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Result of the Pool OpinionWay-Terra Nova, April 12-16 2012

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Majority Judgment Ranking	Majority Grade α	Gauge $+$ or $-$ p ou q	First-Past-the-Post
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2 Bayrou	*Good*	-40.71%	5

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3 Sarkozy	*Acceptable*	$+49.25\%$	2
4 Mélenchon	*Acceptable*	$+42.47\%$	4

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3 Sarkozy	*Acceptable*	+49.25%	2
4 Mélenchon	*Acceptable*	+42.47%	4
5 Dupont-Aignan	*Insufficient*	+40.57%	7
6 Joly	*Insufficient*	−38.53%	6
7 Poutou	*Insufficient*	−45.73%	8

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5 Dupont-Aignan	*Insufficient*	+40.57%	7
6 Joly	*Insufficient*	−38.53%	6
7 Poutou	*Insufficient*	−45.73%	8
8 Le Pen	*Insuffisant*	−47.63%	3

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7 Poutou	*Insufficient*	−45.73%	8
8 Le Pen	*Insuffisant*	−47.63%	3
9 Arthaud	*Insufficient*	−49.93%	9
10 Cheminade	*To Rejetect*	+48.03%	10

Compared to first-past-the-post (plurality voting), majority judgment increases the ranking of moderates and decreases the ranking of the extremes.

Result of the Pool IFOP-La Fabrique Spinoza, April 12-13, 2017

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(1) Mélenchon	*Good*	-35.7%	(1)	20.7%
(2) Macron	*Good*	-41.9%	(3)	20.4%

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(3) Hamon	*Acceptable*	+46.6%	(5)	8.5%
(4) Dupont-Aignan	*Acceptable*	−44.8%	(6)	4.9%
(5) Le Pen	*Acceptable*	−47.7%	(2)	20.5%

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(3) Hamon	*Acceptable*	+46.6%	(5)	8.5%
(4) Dupont-Aignan	*Acceptable*	−44.8%	(6)	4.9%
(5) Le Pen	*Acceptable*	−47.7%	(2)	20.5%
(6) Poutou	*Acceptable*	−48.5%%	(7)	4.2%
(7) Fillon	*Insufficient*	+48.6%	(4)	14.0%
(8) Lassalle	*Insufficient*	+43.6%	(8)	2,1%
(9) Arthaud	*Insufficient*	+42.4%	(9)	1.9%
(10) Asselineau	*Insufficient*	+39.0%	(10)	1.7%
(11) Cheminade	*Insufficient*	+36.8%	(11)	1.1%

Compared to first-past-the-post (plurality voting), majority judgment increases the ranking of moderates and decreases the ranking of the extremes.

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Question asked :

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	Great	*Good*	*Average*	*Poor*	*Terrible*	*Never heard of*
John Kasich	5%	28%	39%	13%	7%	9%
Bernie Sanders	10%	26%	26%	15%	21%	3%
Ted Cruz	7%	22%	21%	17%	19%	4%
Hillary Clinton	11%	22%	20%	16%	30%	1%
Donald Trump	10%	16%	12%	15%	44%	3%

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	p	$\alpha \pm \max\{p, q\}$	q
John Kasich	33%	*Average*+	29%
Bernie Sanders	36%	*Average*−	39%
Ted Cruz	29%	*Average*−	40%
Hillary Clinton	33%	*Average*−	47%
Donald Trump	38%	*Poor*−	47%

Pew Research center poll 2016, Presidential Election, USA

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Clinton :	*Never Great*	*Good*	*Average*	*Poor*	*Terrible*
January	11%	24%	18%	16%	31%
Marsh	11%	22%	20%	16%	31%
August	11%	20%	22%	12%	35%
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This is called the *Condorcet paradox*.

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- A7 [Independence of irrelevant alternatives (IIA)] If $A \succeq B$ then whatever candidates are dropped or adjoined $A \succeq B$.

Impossibility of ranking methods based on comparisons

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Before the performance of Vlascenko, the order was :
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	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	Mark	Place
Urmanov	1	1	1	1	1	2	1	1	1	1/8	1^{st}
Candeloro	3	2	5	2	3	3	5	6	6	3/5	2^{nd}
Zagorodniuk	5	5	4	4	2	4	2	2	3	4/7	3^{rd}
Yagudin	4	3	3	6	4	6	4	3	2	4/7	4^{th}
Kulik	2	4	2	3	6	5	3	4	5	4/6	5^{th}
Vlascenko	6	6	6	5	5	**1**	6	5	4	5/5	6^{th}

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This flip-flop was so strident that the rules used for a century were changed to a new system based on measures similar to one used in gymnastic, in diving and other sport competitions.

The domination paradox

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Merit profile :

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Hollande :	12.5%	16.2%	16.4%	11.7%	14.8%	14.2%	14.2%
Sarkozy :	9.6%	12.3%	16.3%	11.0%	11.1%	7.9%	31.8%

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Possible opinion profile :

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Hollande :	*Exc.*	*V.Good*	*Good*	*Accept.*	*Accept.*	*Poor*	*Rej.*
Sarkozy :	*Outs.*	*Exc.*	*V.Good*	*V.Good*	*Good*	*Accept.*	*Rej.*

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Majority Rule : Sarkozy : 54.3% Hollande : 31.5% Indifferent : 14.2%

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- "What if the minority prefers its alternative much more passionately than the majority prefers a contrary alternative ? Does the majority principle still make sense ?"
- "If there is any case that might be considered the modern analogue to Madison's implicit concept of tyranny, I suppose it is this one."
- To solve the problem, Dahl proposes using an ordinal "intensity scale" obtained "simply by reference to some observable response, such as a statement of one's feelings."

May's and Arrow's axioms

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May's and Arrow's axioms

A method of ranking $\succeq$ is an asymmetric binary relation that compares any two candidates. It must satisfy the following axioms :

- A0$*$ [**Based on measures**] A voter's opinion is expressed by evaluating each candidate in an ordinal scale of grades Γ.
- A1 [Unrestricted Domain] All voter's opinions are admissible.
- A2 [Anonymous] Permuting names of voters does not change the outcome.
- A3 [Neutral] Permuting names of candidates does not change the outcome.
- A4 [Monotone] If $A \succeq B$ and one or more of A's grades are raised then $A \succ B$.
- A5 [Complete] For any two candidates either $A \succeq B$ or $A \preceq B$ (or both, implying $A \approx B$).
- A6 [Transitive] If $A \succeq B$ and $B \succeq C$ then $A \succeq C$.
- A7 [Independence of irrelevant alternatives (IIA)] If $A \succeq B$ then whatever candidates are dropped or adjoined $A \succeq B$.

Ranking methods based on measures

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When does majority rule works well ? (does not have the domination paradox)

Polarization

Definition : an opinion profil is *polarized* between a pair of candidates A and B if for every two voters i and j, if they disagree, they do in opposite directions :

if i evaluates A higher than j, then i evaluates B lower than j.

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Merit profile :

	Out-standing	*Excel-lent*	*Very Good*	*Good*	*Accept-able*	*Poor*	*To Reject*
Hollande :	12.5%	16.2%	16.4%	11.7%	14.8%	14.2%	14.2%
Sarkozy :	9.6%	12.3%	16.3%	11.0%	11.1%	7.9%	31.8%

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Polarized opinion profile :

	12.5%	16.2%	3.1%	7.9%	5.4%	5.7%	6.0%
Hollande :	*Outs.*	*Exc.*	*V.Good*	*V.Good*	*V.Good*	*Good*	*Good.*
Sarkozy :	*Rej.*	*Rej.*	*Rej.*	*Poor*	*Accept.*	*Accept.*	*Good*

	5.0%	9.8%	6.5%	7.7%	4.6%	9.6%
Hollande :	*Accept.*	*Accept.*	*Poor*	*Poor*	*Rej.*	*Rej.*
Sarkozy :	*Good*	*V.Good*	*V.Good*	*Exc.*	*Exc.*	*Outs.*

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Sarkozy :	*Good*	*V.Good*	*V.Good*	*Exc.*	*Exc.*	*Outs.*

Holland : 50.8% Sarkozy : 43.2% Indifferent : 6.0%

Statistical Polarization

True opinion profile, Hollande-Sarkozy, 2012 French presidential poll :

		Hollande							
		Outs.	*Exc.*	*V.G.*	*Good*	*Fair*	*Poor*	*Rej.*	Total
S	*Outs.*	0.14%	0.00%	0.41%	1.09%	2.04%	2.99%	2.99%	09.63%
a	*Exc.*	0.27%	1.09%	0.95%	2.17%	2.71%	2.71%	2.44%	12.35%
r	*V.G.*	0.27%	1.22%	2.04%	3.12%	2.99%	3.93%	2.71%	16.28%
k	*Good*	1.22%	1.09%	1.76%	1.76%	2.85%	1.63%	0.68%	10.99%
o	*Fair*	1.63%	2.44%	2.58%	1.09%	2.31%	0.68%	0.41%	11.13%
z	*Poor*	1.75%	2.58%	1.09%	0.27%	0.54%	0.81%	0.81%	07.87%
y	*Rej.*	7.19%	7.73%	7.60%	2.17%	1.36%	1.49%	4.21%	31.75%
	Total	12.48%	16.15%	16.42%	11.67%	14.79%	14.25%	14.25%	

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Cumulative distibutions of Hollande's grades for each of Sarkozy's grades

		Hollande						
		Outs.	$\preceq$ *Exc.*	$\preceq$ *V.Good*	$\preceq$ *Good*	$\preceq$ *Fair*	$\preceq$ *Poor*	$\preceq$ *Rej.*
S	*Outs.*	01.41%	01.41%	05.64%	16.91%	38.04%	69.03%	100%
a	*Exc.*	02.20%	10.99%	18.68%	36.26%	58.24%	80.23%	100%
r	*V.Good*	01.67%	09.17%	21.67%	40.84%	59.17%	83.34%	100%
k	*Good*	11.11%	20.99%	37.04%	53.09%	79.02%	93.83%	100%
o	*Fair*	14.63%	36.58%	59.75%	69.51%	90.24%	96.34%	100%
z	*Poor*	22.41%	55.17%	68.96%	72.41%	79.31%	89.65%	100%
y	*Rej.*	22.65%	47.01%	70.94%	77.78%	82.05%	86.75%	100%

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When the language of grades is sufficiently rich, a method of ranking based on measures satisfying basic axioms A1 to A7 is consistent with the majority rule on polarized pairs if and only if it coincide with the majority-gauge.

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Because it is in this situation that voters have the greatest temptation to vote strategically and MR is stable to strategic voting.

Strategy proofness for methods based on measures

- A1 [Unrestricted Domain] All voters opinions are admissible.
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In our field experiments, 4 grades were few, 6 grades were sufficient

No. of grades :	1	2	3	4	5	6	7	Total
2007 :	1%	2%	10%	31%	42%	14%	–	100%
2012 :	1%	6%	13%	31%	36%	13%	1%	100%

Poll Opinion Way/Terra Nova, French presidential, April 12-16, 2012

Condorcet-ranking	Hollande	Bayrou	Sarkozy	Mélenchon	Le Pen	Borda-ranking
1 Hollande	–	51.6%	53.9%	68.5%	64.1%	1) 59.5%
2 Bayrou	48.4%	–	56.5%	59.4%	70.5%	2) 58.7%
3 Sarkozy	46.1%	43.5%	–	50.5%	65.7%	3) 51.4%
4 Mélenchon	31.5%	40.6%	49.5%	–	59.7%	4) 45.3%
5 Le Pen	35.9%	29.5%	34.3%	40.3%	–	5) 35.0%

Poll Opinion Way-Terra Nova, French presidential, April 12-16, 2012.

Majority judgment	Majority grade	Gauge	First-past-the-post		Approval Voting	
1 Hollande	*Good*	+45.1%	1	28.6%	1	49.4%
2 Bayrou	*Good*	−40.7%	5	9.1%	3	39.2%
3 Sarkozy	*Accept*	+49.3%	2	27.3%	2	40.5%
4 Mélenchon	*Accept*	+42.5%	4	11.0%	4	39.1%
5 Dupont-Aignan	*Poor*	+40.6%	7	1.5%	8	10.7%
6 Joly	*Poor*	−38.5%	6	2.3%	6	26.7%
7 Poutou	*Poor*	−45.7%	8	1.2%	7	13.3%
8 Le Pen	*Poor*	−47.6%	3	17.9%	5	27.4%
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- Methods that ask more information (MJ-, Condorcet and Borda) have identical rankings and put Bayrou comfortably ahead of Sarkozy.
- Methods that ask less information (first-past-the-post and AV) fail.

Common use of grades : Orsay experiment, 2007

	3 prcts.	1^{st} prct.	6^{th} prct.	12^{th} prct.	Samples of 100 Avg. (σ)	Rg	Dsjt samples of 50 Avg. (σ)	Rg
Excll	0.7	0.7	0.7	0.7	0.7 (.07)	0.6/0.8	0.7 (.12)	0.5/0.9
V.Good	1.3	1.2	1.2	1.4	1.2 (.13)	1.1/1.5	1.3 (.16)	1.1/1.5
Good	1.5	1.5	1.4	1.6	1.5 (.13)	1.4/1.7	1.5 (.27)	0.9/1.8
Accp	1.7	1.7	1.7	1.8	1.8 (.15)	1.7/2.1	1.7 (.27)	2.1/2.6
Poor	2.3	2.3	2.3	2.2	2.3 (.19)	2.1/2.7	2.3 (.19)	2.1/2.6
Rjct	4.6	4.8	4.6	4.3	4.5 (.29)	4.1/4.8	4.5 (.41)	4.1/5.3

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V.Good	1.3	1.2	1.2	1.4	1.2 (.13)	1.1/1.5	1.3 (.16)	1.1/1.5
Good	1.5	1.5	1.4	1.6	1.5 (.13)	1.4/1.7	1.5 (.27)	0.9/1.8
Accp	1.7	1.7	1.7	1.8	1.8 (.15)	1.7/2.1	1.7 (.27)	2.1/2.6
Poor	2.3	2.3	2.3	2.2	2.3 (.19)	2.1/2.7	2.3 (.19)	2.1/2.6
Rjct	4.6	4.8	4.6	4.3	4.5 (.29)	4.1/4.8	4.5 (.41)	4.1/5.3

The use of language was common in the 3 percents, yet, the majority judgement winner was not the same.

Favoring the centrist, Orsay experiment 2007

10,000 random samples of 201 ballots from 501 "representative" ballots

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	Left ⟵		⟶ *Right*		
	Royal	Bayrou	Sarkozy	Tie	Cycle
First-past-the-post winner	977	0	9,022	5	–
Two-past-the-post winner	1,146	98	8,197	559	–
Approval $\succeq$ *Very Good*	467	658	7,947	928	–
Majority judgement-winner	606	4,326	5,065	3	–
Condorcet-winner	142	8,329	974	441	114
Approval $\succeq$ *Good*	23	9,465	40	472	–
Point-summing	139	9,463	239	159	–
Borda-winner	12	9,976	0	12	–

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Borda-winner	12	9,976	0	12	–

First- and two-past-the-post (unduly) penalize the centrist, point-summing and Borda (unduly) favor the centrist.

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Strategic manipulability, Orsay experiment 2007

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Numbers of successful strategic manipulations :

	Point-sum	Borda	First-p-p	Approval $\succeq$ *Good*	Approval $\succeq$ *VGood*	Cond-orcet	Majority judge
Strat 1	9,965	9,313	8,699	8,569	8,407	7,042	6,142
Strat 2	9,769	7,864	4,411	8,849	8,557	4,641	5,313

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No method elects the Condorcet-winner as a Nash or strong equilibrium with honest votes. With majority judgement, there are strong-equilibria where the Condorcet winner is elected with his true majority grade and the majority of grades received a candidate are honest.

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- It has also been proposed to the Special Committee on Electoral Reform in Quebec City by the deputy Raymond Côté, in September 22, 2016.

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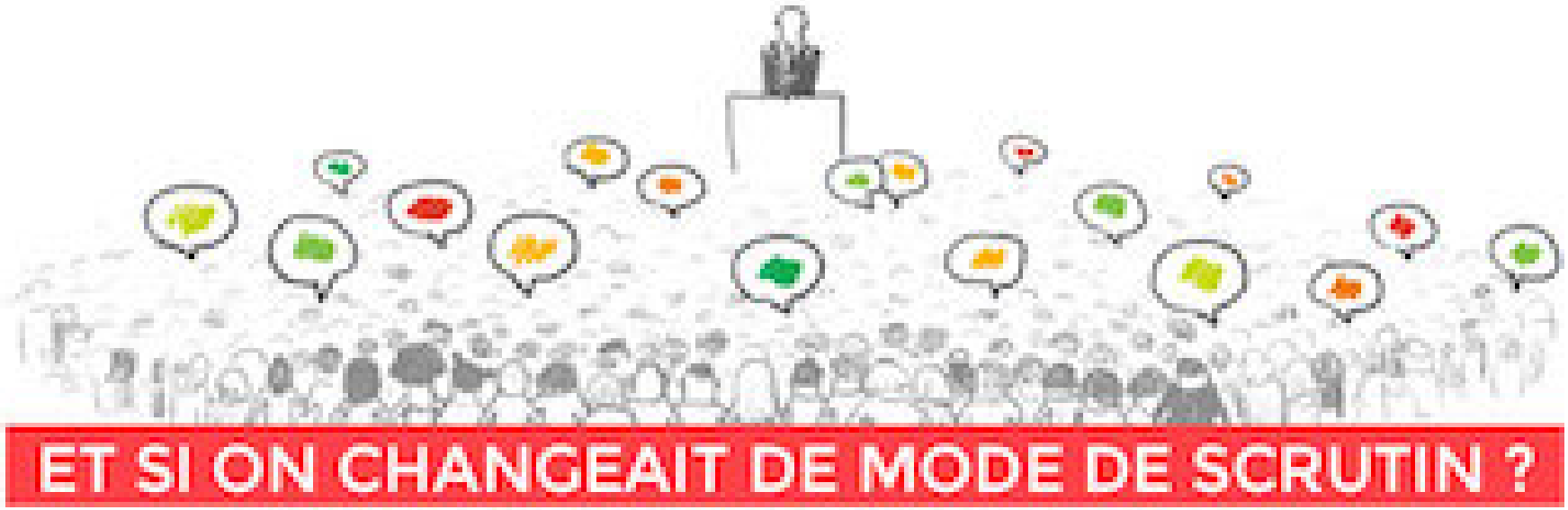

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